

# Production of Commodities by means of Commodities

Piero Sraffa, 1960

As a “Prelude to a Critique of Economic Theory”, Sraffa explains the price system of a static economy. The demonstration, albeit not formalized, is rigorous and constructivist.

## I. Production for Subsistence

In a self-replacing economy (when the production allows to just reproduce its means of production), there is only one price system possible.

## II. Production with a Surplus

When the economy produces more than it needs to replicate, the profit must be equalized among sectors. This writes  $(1+r) \cdot Z = Y$ , where  $Z$  is the transformation matrix and  $Y$  the gross output (i.e. part of  $Y$  is consumed in the production process  $Z$ ). A *basic* product (as opposed to *non-basic* or *luxury*) is a commodity used to produce any other commodity (directly or not). Wages are excluded from the transformation matrix as part of them is spent arbitrarily in *non-basic* products. The equation rewrites  $(1+r) \cdot Z + w \cdot L = Y$ , where  $w$  is the wage and  $L$  is the vector of labor required for each commodity. The national income is the surplus, i.e. the part of  $Y$  that is not used in the production.

## III. Proportions of labour to means of production

The national income is normalized to 1, which is thus the maximum wage. Variations in the wage from 0 to 1 imply variations in prices, as sectors with a higher (resp. lower) share of labour in their means of production relative to other commodities need to lower (resp. increase) their price when wage increases so as to offset the deficit (resp. surplus).

## IV. The standard commodity

A unique composite commodity is *standard*, i.e. its price doesn't vary with wage (as the means of production recursively required to produce it are in the same proportion as its components). It is used as numéraire to express prices. The standard ratio  $R$  of net product to means of productions (

$R = \frac{Y - M}{M}$ , where  $Y$  and  $M$  are expressed in value) is also equal to the maximum rate of profit, so that  $r = R \cdot (1 - w)$ .

**Note:** Tucci (1976) formalizes the Sraffian theory and shows that the price vector is

$p = w(I - (1+r)Z)^{-1} \cdot L$ . The relation between  $r$  and  $w$  is solved using  $R = \frac{1 - \lambda}{\lambda}$ , where  $\lambda$  is the Perron and Frobenius eigenvalue of  $Z$ . More directly, given  $r \in (0; 1)$ , the solution is  $w = (e \cdot (I - Z) \cdot (I - (1+r)Z)^{-1} \cdot L)^{-1}$ , where  $e$  is the row vector of ones (which sums across columns).

## V. Uniqueness of the standard system

## VI. Reduction to dated quantities of labour

The cost of production of a commodity  $A$  can be expressed as a function of dated labour:  $p_a A = \sum_n L_{a,n} w (1+r)^n$ . That expression is the labour embodied in  $A$  when  $r=0$ . The relative weight of  $w$  and  $r$  vary at different dates, unless  $r=0$ . The term which reaches its maximum has the 'date'  $n = \frac{1+r}{R-r}$ . Only for the most recent dates does the cost decrease with the rate of profit. Sraffa concludes at the impossibility of aggregating 'periods' belonging to several terms of labour into a single magnitude which could be regarded as capital. Hence capital cannot be a measurable quantity independent of distribution and prices.

**Pb:** Sraffa talks of dates whereas the chronological steps need not be yearly.

## Part 2: Multiple-Product Industries and Fixed Capital

### VII. Joint Production

Coproduction in a single process is introduced, so that prices are no more determined (because there are more variables than equations). Thus, Sraffa assumes that there are as many different processes as there are commodities, i.e. that different technologies coexist to produce coproducts. An industry is now characterised by the proportions in which it uses and the proportions in which it produces the various commodities.

### VIII. The Standard System with Joint Products

Negative multipliers appear in the definition of the standard commodity: the intuition is lost, but the equations remain valid. Three concentric types of *non-basics* are distinguished: products which don't enter the means of production of any industry, those which enter only its own means of production, those which only enter the means of production of an interconnected group of non-basics (i.e. products that behave as non-basics as a group).<sup>1</sup>

### IX. Other Effects of Joint Production

The labour 'reduction' approach of chapter VI. is inapplicable to commodities in general, but remains valid for the system as a whole. Prices of coproducts can be negative. We can no longer speak of a rise or a fall in wage unless we specify the standard, for what is a rise in one standard may be a fall in another. Contrarily to single-product industries, a same wage in terms of a given commodity can be attained by several rates of profit. However, wages are uniquely determined by the rate of profit.

### X. Fixed Capital

Durable instruments of production are part of the intake of a process, and the portion that is not used up will also be treated as the joint product of the industry (hence joint production is the norm rather than an exception). This convention for depreciation allows to correctly attribute a uniform rate of profits. A discussion on how to account for depreciation ensues. Notably, it shows that the book value of an old machine depreciates less rapidly in the first years and more rapidly in the last with a higher rate of profit, so that its value is always higher. Don't really understand why, but a system with no other element of joint production besides fixed capital has an all-positive standard commodity.

### XI. Land

Natural resources in short supply occupy a position equivalent to *non-basics*. As such, taxing them will not impact prices nor the rate of profit (but only their owner). In fact, the complications of the joint production arise as soon as one commodity is produced by more than one method.

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<sup>1</sup> The latter notion seems linked to the reducibility or the rank of the matrix of transformation.

### Part 3, XII.: Switch in Methods of Production

As the rate of profit varies, one method or another to produce the same *non-basic* commodity might be the least costly, i.e. the one chosen. In the case of *basic* commodities, the price system and the maximum rate of profit depend on which method is chosen, so one cannot compare their cost on a common basis. The trick is to choose one system and treat the alternative method as a *non-basic* product: then, whatever the system chosen, the cost of both method will follow the same order for the highest rates of profit (the cheapest is eventually chosen, and it corresponds to the case where  $R$  is the highest when that method is *basic*). Although switching from one method to another happens frequently as the rate of profit moves, and correspond to a switch from one system to another (and their corresponding standard commodities), a rise in the rate of profit invariably corresponds to a fall in the wage measured in terms of any commodity. The chapter ends by treating the case of switch in methods of production (between competing method) in the multiple-product case.

The book has no conclusion, but four appendices: A on 'sub-systems', B on 'self-reproducing *non-basics*', C on the device of a 'basic system' and D: references to the literature.